APPENDIX

A.1 MMH: Max-margin Harmonium

For the special max-margin Harmonium (MMH), the learning problem is the same as defined in Section 4.1.1, and only several changes are needed to estimate parameters based on the general learning procedure. In this section, we present the necessary changes for learning MMH. For any other special cases of multi-view Markov networks, the learning can be similarly done.

With the definitions of local conditionals in Section 3.1, we can directly write the joint model distribution $p(\mathbf{x}, \mathbf{z}, \mathbf{h})$ based on the constructive definition and the marginal data likelihood $p(\mathbf{x}, \mathbf{z})$

$$p(\mathbf{x},\mathbf{z}) \propto \exp\Big\{\alpha^\top\mathbf{x}+\beta^\top\mathbf{z}-\frac{1}{2}\sum_j\frac{z_j^2}{\sigma_j^2}+\frac{1}{2}\sum_k(\mathbf{x}^\top\mathbf{W}_{\cdot k}+\mathbf{z}^\top\mathbf{U}_{\cdot k})^2\Big\}.$$

Then, we use the contrastive divergence method and introduce two variational distribution q_0 and q_1. In this case, we can make a superficially simpler mean field assumption that $q(\mathbf{x},\mathbf{z},\mathbf{h}) = \prod_i q(x_i)\prod_j q(z_j)\prod_k q(h_k)$. Indeed, the general structured mean field assumption as made in Section 4.1.2 will lead to the same results, that is, a fully factorized form of $q(\mathbf{x})$, $q(\mathbf{z})$ and $q(\mathbf{h})$. Specifically, we have the following fully factorized update rules for posterior inference of q

$$\begin{aligned} q(\mathbf{x}) &= \prod_i q(x_i) = \prod_i p(x_i|\mathbb{E}_{q(\mathbf{h})}[\mathbf{h}]) \\ q(\mathbf{z}) &= \prod_j q(z_j) = \prod_j p(z_j|\mathbb{E}_{q(\mathbf{h})}[\mathbf{h}]) \\ q(\mathbf{h}) &= \prod_k q(h_k) = \prod_k p(h_k|\mathbb{E}_{q(\mathbf{x})}[\mathbf{x}], \mathbb{E}_{q(\mathbf{z})}[\mathbf{z}]). \end{aligned}$$

Similarly, (x_i, z_j) are clamped at their observed values for q_0, and only $q(h_k)$ is updated. The distribution q_1 is achieved by performing the above updates starting from q_0. Several iterations can yield a good q_1. After we have inferred q_0 and q_1, parameter estimation can be done by an alternating procedure as in Section 4.1.2, where the step of estimating $\mathbf{V}$ with Θ fixed is to learn a multi-class SVM

$$\min_{\mathbf{V}} \ \frac{1}{2}C_1\|\mathbf{V}\|_2^2 + C_2\sum_d \max_y[\Delta\ell_d(y) - \mathbf{V}^\top\mathbb{E}_{p(\mathbf{h}|\mathbf{x}_d,\mathbf{z}_d)}[\Delta\mathbf{f}_d(y)]].$$

Note that in this case, the latent representation (i.e., expectation of $\mathbf{H}$) is simply written as $\mathbb{E}_{p(\mathbf{h}|\mathbf{x},\mathbf{z})}[\mathbf{h}] = \Upsilon$, where $\Upsilon_k = \mathbf{x}^\top\mathbf{W}_{\cdot k} + \mathbf{z}^\top\mathbf{U}_{\cdot k}, \ \forall 1 \leq k \leq K$, when input data $\mathbf{x}$ and $\mathbf{z}$ are fully observed. If missing values exist in $\mathbf{x}$ or $\mathbf{z}$, the corresponding components are replaced with their expected values. Therefore, the prediction tasks (e.g., classification and retrieval) can be easily done in testing, as detailed in Section 4.4.

For the step of estimating Θ, the sub-gradient is computed as

$$\begin{aligned} \partial\alpha_i &= -\mathbb{E}_{q_0}[x_i] + \mathbb{E}_{q_1}[x_i], \\ \partial\beta_j &= -\mathbb{E}_{q_0}[z_j] + \mathbb{E}_{q_1}[z_j], \\ \partial(\sigma_j^{-1}) &= -\mathbb{E}_{q_0}[z_j^2\sigma_j^{-1}] + \mathbb{E}_{q_1}[z_j^2\sigma_j^{-1}], \\ \partial\mathbf{W}_{ik} &= -\mathbb{E}_{q_0}[x_i h'_k] + \mathbb{E}_{q_1}[x_i h'_k] - C_2\sum_d(\mathbf{V}_{y_d k} - \mathbf{V}_{\bar{y}_d k})\mathbb{E}_{q_0}[x_i] \\ \partial\mathbf{U}_{jk} &= -\mathbb{E}_{q_0}[z_j h'_k] + \mathbb{E}_{q_1}[z_j h'_k] - C_2\sum_d(\mathbf{V}_{y_d k} - \mathbf{V}_{\bar{y}_d k})\mathbb{E}_{q_0}[z_j], \end{aligned}$$

where $h'_k = \mathbf{x}^\top\mathbf{W}_{\cdot k} + \mathbf{z}^\top\mathbf{U}_{\cdot k}$ and $\bar{y}_d = \arg\max_y[\Delta\ell_d(y) + \mathbf{V}^\top\mathbb{E}_{q_0}[\mathbf{f}(y, \mathbf{h}_d)]]$ is the *loss-augmented prediction*. Based on the definition of q_0, the expectations $\mathbb{E}_{q_0}[x_i]$ and $\mathbb{E}_{q_0}[z_j]$ are actually the count frequency of x_i and z_j, respectively.

A.2 Multi-view Latent Subspace MN for Modeling Paragraph Ordering Information

In this section, we formally define the structured multi-view latent subspace Markov network used in Section 5.4. Let $\mathbf{x}$ be an $P \times N$ observation matrix, where P is the number of paragraphs in a document and N is the vocabulary size. Each row $\mathbf{x}_p$ is a vector, of which the element $x_{pi} = 1$ if word i appears in paragraph p; otherwise $x_{pi} = 0$. Each column $\mathbf{x}_{\cdot i}$ represents the appearance pattern of word i in all paragraphs. To consider the paragraph ordering information, we define a first-order Markov chain on each $\mathbf{x}_{\cdot i}$ while assuming that different $\mathbf{x}_{\cdot i}$'s are conditional independent. More formally, we define the factorial conditional distribution $p(\mathbf{x}|\mathbf{h}) = \prod_{i=1}^N p(\mathbf{x}_{\cdot i}|\mathbf{h})$, where each $p(\mathbf{x}_{\cdot i}|\mathbf{h})$ is a linear chain CRF [27]. This model is actually an N-view latent subspace MN, where the ith view has P variables $\{X_{pi}\}_{p=1}^P$ that are connected via a linear chain.

By the constructive definition as discussed in Section 3 (please see Eq. (1) and Eq. (2)), we need to specify the edge set for each view and define the feature functions. Specifically, let E denote the set of generalized edges (including both singleton vertices and pairwise edges) of a linear chain on each view[13], then $E = \{(1), \cdots, (P), (1,2), \cdots, (P-1, P)\}$, where (p) is a degenerate edge (i.e., the node p). Accordingly, the feature functions ϕ consists of a *singleton* feature function g that is defined on a single variable and 4 *pairwise* feature functions ϕ_0, ϕ_1, ϕ_2 and ϕ_3 that defined on a pair of variables. Mathematically, we define the singleton feature function as

$$g(x_{pi}) = x_{pi},$$

and we define the 4 pairwise feature functions as

$$\forall j = 0, \cdots, 3: \ \ \phi_j(x_{pi}, x_{p+1,i}) = \begin{cases} 1, & \text{if } 2x_{pi} + x_{p+1,i} = j \\ 0, & \text{otherwise} \end{cases}$$

We denote the corresponding weights by α and $\beta_j, \ j = 0, \cdots 3$. To make the model reasonably rich, we assume

[13] By definition, all views have the same set of edges

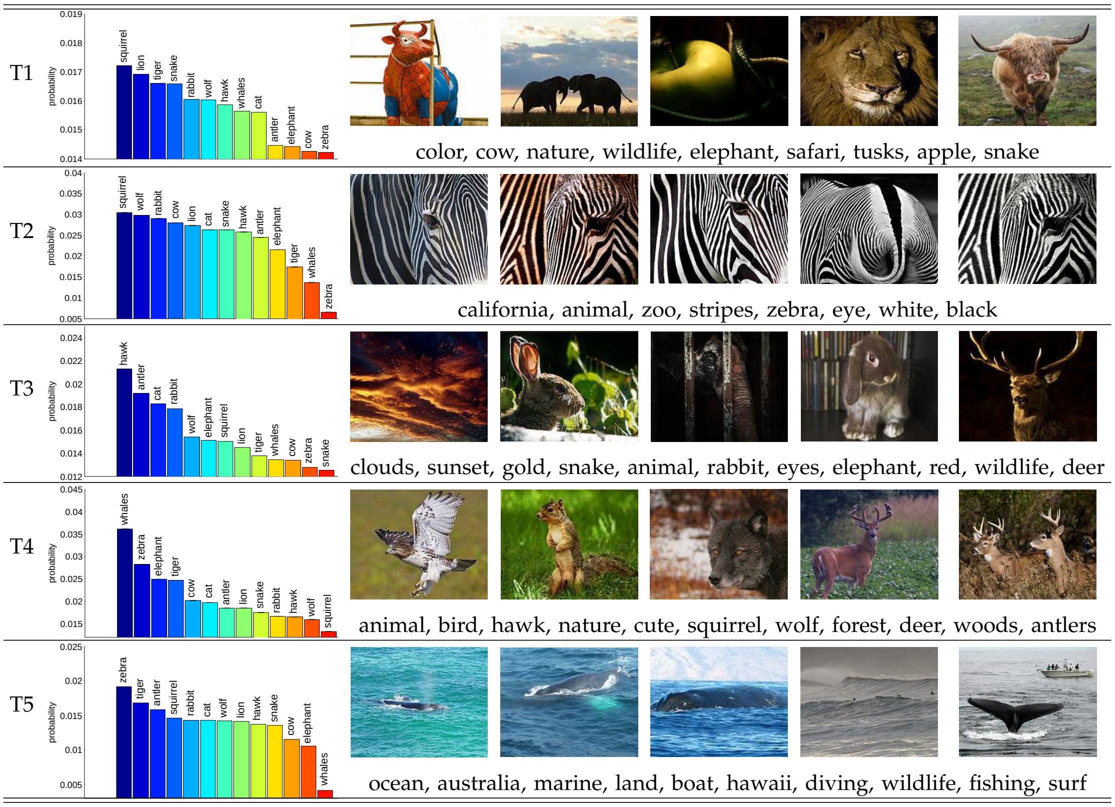


Fig. 11. Example topics discovered by the 60-topic MMH as in Fig. 3 on the Flickr animal dataset. For each topic, we show 5 bottom-ranked images as well as the average probabilities of that topic on representing images from the 13 categories.

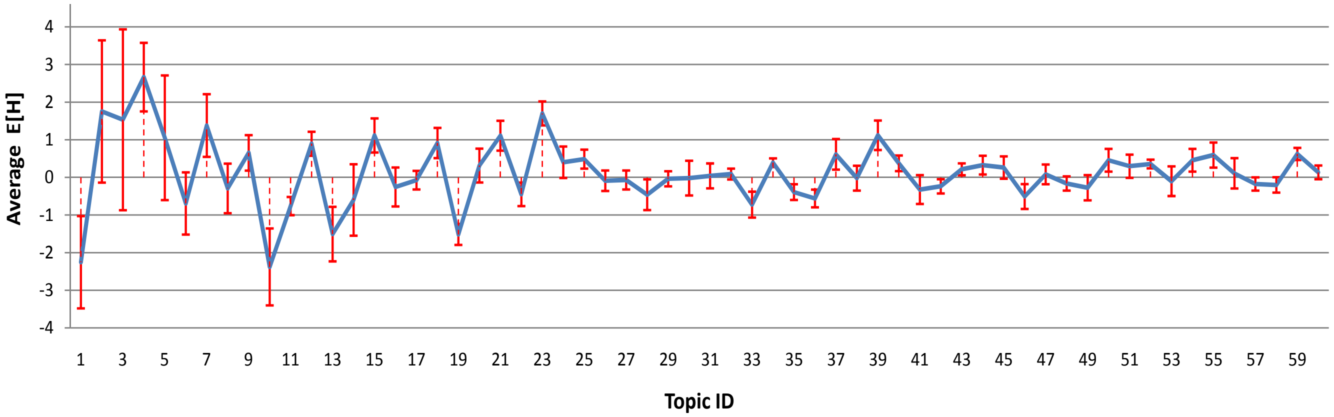


Fig. 12. Average value and variance of E[H] discovered by a 60-topic MMH across 13class-animal Flickr images.

different views have different feature weights, while within each view, the weights of these feature functions are shared by all edges. For notation simplicity, we define $g(\mathbf{x}_{\cdot i}) \triangleq \sum_{p=1}^{P} g(x_{pi})$, which is the accumulated function value of g evaluated on the sequence $\mathbf{x}_{\cdot i}$, and $\phi_j(\mathbf{x}_{\cdot i}) \triangleq \sum_{p=1}^{P-1} \phi_j(x_{pi}, x_{p+1,i})$, which is again an accumulated function value of ϕ_j. Now, we define the interaction terms between $\mathbf{X}$ and latent variables $\mathbf{H}$ as

$$\sum_{i=1}^{N} (g(\mathbf{x}_{\cdot i}) \mathbf{U}^i \mathbf{h} + \sum_j \phi_j(\mathbf{x}_{\cdot i}) \mathbf{W}_j^i \mathbf{h}),$$

where $\mathbf{W}_j^i$ and $\mathbf{U}^i$ are K-dimensional real vectors. Finally, we include a quadratic energy term of the real variables $\mathbf{H}$ in the exponent of the joint distribution.

Putting the above definitions together, we define the joint distribution

$$p(\mathbf{x}, \mathbf{h}) \propto \exp\Big\{ \sum_i g(\mathbf{x}_{\cdot i})(\alpha^i + \mathbf{U}^i \mathbf{h}) + \sum_{ij} \phi_j(\mathbf{x}_{\cdot i})(\beta_j^i + \mathbf{W}_j^i \mathbf{h}) - \frac{1}{2} \mathbf{h}^\top \mathbf{h} \Big\},$$

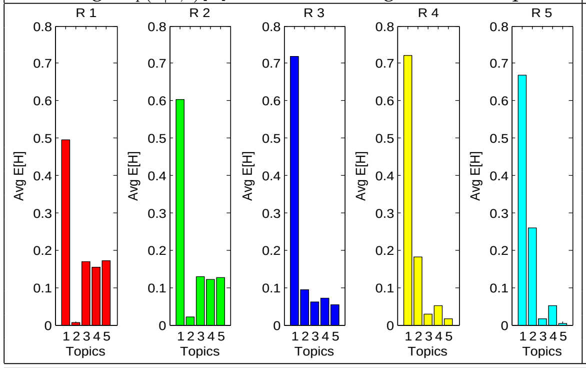
and we have the conditional distributions as

$$p(\mathbf{x}_{\cdot i} | \mathbf{h}) \propto \exp\Big\{ g(\mathbf{x}_{\cdot i})(\alpha^i + \mathbf{U}^i \mathbf{h}) + \sum_j \phi_j(\mathbf{x}_{\cdot i})(\beta_j^i + \mathbf{W}_j^i \mathbf{h}) \Big\}$$

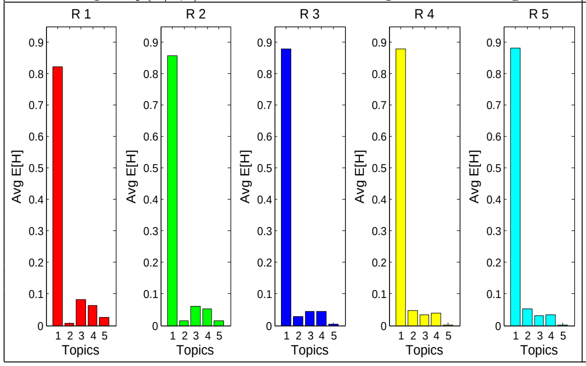
$$p(h_k | \mathbf{X}) = \mathcal{N}(h_k | \sum_{i=1}^{N} (g(\mathbf{x}_{\cdot i}) \mathbf{U}_k^i + \sum_j \phi_j(\mathbf{x}_{\cdot i}) \mathbf{W}_{jk}^i), 1).$$

TABLE 2
Average distributions over the topics for documents with different rating scores by a 5-topic MMH, 5-topic TWH and 5-topic DWH.

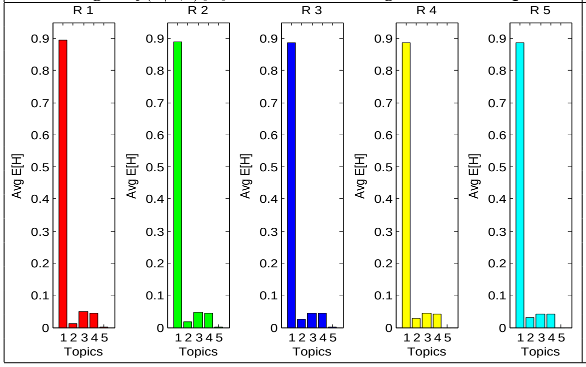
Max Margin Harmonium (Avg-KL: 3.568)

Average $\mathbb{E}_{p(\mathbf{h}\mid\mathbf{x},\mathbf{z})}[\mathbf{h}]$ in 5-level Rating Score examples	T1	T2	T3	T4	T5
	room	*great*	small	worst	worst
	hotel	*loved*	worst	small	dirty
	n't	arrived	dirty	dirty	small
	time	*enjoyed*	shower	shower	shower
	stay	*fantastic*	broken	broken	broken
	day	bit	smell	smell	smell
	night	*wonderful*	paying	paying	paying
	good	*lovely*	bathroom	bathroom	poor
	staff	pool	poor	poor	toilet
	pool	trip	toilet	toilet	refund
	back	beach	staying	refund	manager
	rooms	*fun*	refund	staying	bathroom
	food	*happy*	breakfast	walls	walls
	area	pools	hotel	hotel	carpet
	nice	*perfect*	walls	carpet	paid

Tri-Wing Harmonium (Avg-KL: 0.045)

Average $\mathbb{E}_{p(\mathbf{h}\mid\mathbf{x},\mathbf{z})}[\mathbf{h}]$ in 5-level Rating Score examples	T1	T2	T3	T4	T5
	room	beach	bathroom	resort	experience
	hotel	food	parking	beach	breakfast
	n't	pool	tv	ocean	stay
	time	*great*	area	trip	service
	stay	bar	coffee	vacation	*beautiful*
	day	resort	kitchen	desk	dinner
	night	restaurants	bed	check	guests
	good	drinks	street	time	made
	staff	restaurant	floor	front	*wonderful*
	pool	view	*large*	*great*	feel
	back	lunch	small	called	trip
	rooms	*good*	tub	call	house
	food	sea	*comfortable*	property	visit
	area	*beautiful*	location	*beautiful*	special
	nice	walk	internet	people	*comfortable*

Dual-Wing Harmonium (Avg-KL: 0.038)

Average $\mathbb{E}_{p(\mathbf{h}\mid\mathbf{x},\mathbf{z})}[\mathbf{h}]$ in 5-level Rating Score examples	T1	T2	T3	T4	T5
	room	beach	food	breakfast	belize
	hotel	food	told	reception	brett
	n't	pool	asked	bathroom	cam
	time	resort	holiday	bed	canapes
	stay	*great*	reception	shower	canoeing
	day	restaurants	day	holiday	caracol
	night	bar	bar	coffee	hosts
	good	drinks	staff	evening	nadege
	staff	restaurant	back	small	underway
	pool	lunch	manager	*clean*	wineries
	back	sea	people	bar	adopted
	rooms	*beautiful*	evening	hotel	amanda
	food	entertainment	entertainment	*good*	aurora
	area	pools	arrived	main	begun
	nice	view	hotel	tea	boasted

Now, we can follow the procedure in Section 4 to perform parameter estimation and inference. Sine each view is a linear-chain Markov network, we can perform inference with a forward-backward message passing scheme, which can be done in the same way as in [27]. We omit the details for brevity.

The message passing for each document is of a complexity $O(N \times P \times S^2)$, where S is the number of possible values for each variable X_{pi}. Since X_{pi} is binary, we have $S^2 = 4$ (a very small constant). Moreover, the number of paragraphs P is on average very small, e.g., the average P is 9 in the hotel review dataset. Therefore, the time complexity is linear in terms of the feature dimension N (i.e., the number of terms in the given vocabulary).

A.3 Additional Experimental Results

Fig. 11 shows the 5-bottom ranked (i.e., with small expectation values of H_k) images for each of the 5 topics, as presented in Fig. 3. For convenience, we also include the average probability of each category distributed on the particular topic, which is the same as that in Fig. 3.

Fig. 12 shows the average value of $\mathbb{E}[\mathbf{H}]$ discovered by a 60-topic MMH, together with the variance across the images from the 13 classes of animals. The variance of each $\mathbb{E}[H_k]$ indicates the discriminative power of topic k over all the 13-class images.

Table 2 shows the complete table which contains the average distributions over topics for all the three methods, i.e., MMH, TWH, and DWH.